– failure of the timber adjacent to the glue line;
– failure of the timber member (e.g. pull-out of a whole timber block with several glued-in rods).

The first failure mode is already considered in the strength of the equivalent T-stub in tension (mode 3), whereas the other brittle modes have to be avoided, for example by designing the connection according to Ref. [5]

Design Stiffness of Basic Joint Components

The stiffness coefficients k_i (*Fig. 4*) can be determined in accordance with Ref. [12].

(i) *The stiffness coefficient for end-plate in bending under tension* (for a single bar-row) is:

– with prying forces:

$$k_p = \frac{0{,}85 \cdot l_{eff,t} \cdot t_p^3}{m^3} \qquad (9)$$

– without prying forces:

$$k_p = \frac{0{,}425 \cdot l_{eff,t} \cdot t_p^3}{m^3} \qquad (10)$$

where $l_{eff,t}$, t_p and m are the effective length, the thickness and a geometrical parameter, respectively, of the T-stub flange (*Fig. 5*).

(ii) *The stiffness coefficient for bars in tension* (for a single bar-row) is:

– with prying forces:

$$k_b = \frac{1{,}6 \cdot A_s}{L_b} \qquad (11)$$

– without prying forces:

$$k_b = \frac{2{,}0 \cdot A_s}{L_b} \qquad (12)$$

where A_s is the effective bar area and L_b is the steel bar elongation length, taken as equal to the sum of α times the nominal bar diameter, the plates thickness, the washer and half of the height of the nut. The parameter α previously introduced can be determined taking into account the Volkersen analysis of a single lap joint.[14] For the case of an axial-symmetric joint (*Fig. 7*), according to Volkersen, it is possible to write the following four equations (global equilibrium in x, indefinite equilibrium

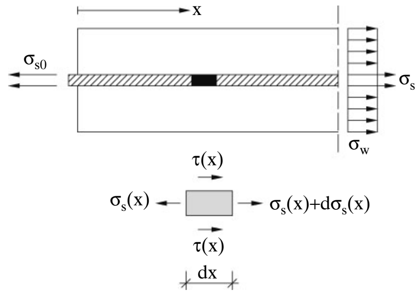

Fig. 7: Axial-symmetric joint with a glued-in rod

for the element dx, indefinite compatibility and constitutive law equation):

$$\sigma_{s0} \cdot A_s = \sigma_s(x) \cdot A_s + \sigma_w(x) \cdot A_w \qquad (13)$$

$$d\sigma_s(x) \cdot A_s = -\pi \cdot \Phi \cdot \tau(x) \cdot dx \qquad (14)$$

$$\frac{ds(x)}{dx} = \varepsilon_w(x) - \varepsilon_s(x) = \frac{\sigma_w(x)}{E_w} - \frac{\sigma_s(x)}{E_s} \qquad (15)$$

$$s(x) = \frac{t}{G} \cdot \tau(x) \qquad (16)$$

where t is the thickness of the glue line, s is the slip between the adherents at glue line level, Φ is the diameter of the bar, G is the shear modulus of the glue.

After few operations it is possible to obtain the following differential equation:

$$\frac{d^2\tau(x)}{dx^2} = \tau(x) \cdot \frac{G \cdot \pi \cdot \Phi}{t} \cdot \left[\frac{1}{A_s \cdot E_s} + \frac{1}{A_w \cdot E_w}\right] \qquad (17)$$

Adopting the following positions:

$$\psi = \frac{E_s \cdot A_s}{E_w \cdot A_w} \quad \text{[adimensional]} \qquad (18)$$

$$\Gamma = \frac{G \cdot \pi \cdot \Phi}{E_s \cdot A_s \cdot t} \quad [1/L^2] \qquad (19)$$

$$\omega^2 = \Gamma \cdot (1+\psi) \quad [1/L^2] \qquad (20)$$

the equation can be written as:

$$\frac{d^2\tau(x)}{dx^2} = \tau(x) \cdot \omega^2 \qquad (21)$$

Solving the equation, after some manipulations it is possible to express the elongation of the initial part of the bar (where τ decreases from the maximum value at the timber surface to 0):

$$\Delta = \frac{\sigma_{s0}}{E_s} \cdot \frac{1}{(1+\psi) \cdot \omega} \qquad (22)$$

To calculate the stiffness of the glued-in rods in tension, an equivalent length L_b can be defined:[13]

$$\Delta = \frac{\sigma_{s0}}{E_s} \cdot L_b = \frac{\sigma_{s0}}{E_s} \cdot (\alpha \cdot \Phi) \qquad (23)$$

The following value for α can then be obtained:

$$\alpha = \frac{L_b}{\Phi} = \frac{1}{(1+\psi) \cdot \omega \cdot \Phi} \qquad (24)$$

According to expression (24), the parameters affecting the value of α are as follows: the wood elastic modulus E_w, the steel elastic modulus E_s, the diameter Φ of the steel bar, the timber area A_w (assumed equal to 36 Φ^2), the thickness t and the shear modulus G of the glue line. *Figure 8* shows, as an example, the range of variability for the parameter α assuming t = 2 mm and G = 1,5 MPa.

(iii) *The stiffness coefficient for timber in compression* is defined as:

$$k_t = \frac{E_w \cdot \sqrt{b_{eff,c} \cdot l_{eff,c}}}{\beta \cdot E_s} \qquad (25)$$

where E_w is the wood elastic modulus parallel to the grain; E_s is the steel elastic modulus; $b_{eff,c}$ and $l_{eff,c}$ are the effective dimensions of the flange of the equivalent T-stub in compression (see *Fig. 9*).

To obtain the value of the coefficient β, the approach proposed in Ref. [13] for the case of concrete in compression, is adopted. The stiffness of a rigid plate supported by an elastic half space could be considered according to Ref. [15]:

$$K_z = \frac{P}{\delta_r} = \frac{G}{1-\upsilon} \cdot \beta_z \cdot \sqrt{b_{eff,c} \cdot l_{eff,c}} \qquad (26)$$

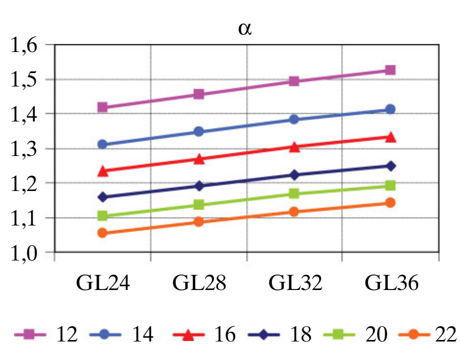

Fig. 8: Range of the values of parameter α